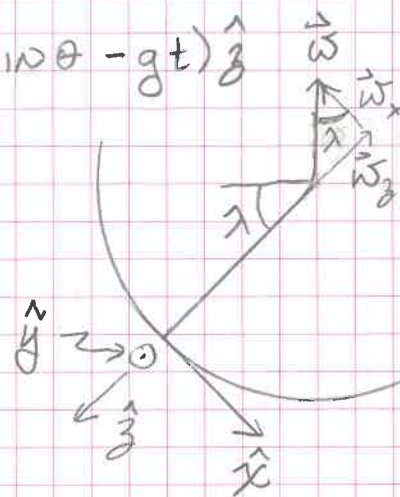
TM5 10.18

A BRITISH WARSHIP FIRES A PROJECTILE DUE SOUTH AT 50°S. IF THE SHELLS ARE FIRED AT 37° AND $v = 800\ m/s$, BY HOW MUCH DO THEY MISS THEIR TARGET?

USE COORDINATES SHOWN

$$\Rightarrow \vec{v} = v_0 \cos\theta\, \hat{x} + 0\,\hat{y} + (v_0 \sin\theta - gt)\,\hat{z}$$

$$\vec{\omega} = -\omega\cos\lambda\, \hat{x} - \omega \sin\lambda\, \hat{z}$$



TAKE THE CROSS PRODUCT

$$\vec{\omega}\times\vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -\omega\cos\lambda & 0 & -\omega\sin\lambda \\ v_0\cos\theta & 0 & v_0\sin\theta - gt \end{vmatrix}$$

$$= 0\,\hat{x} - \left[(-\omega\cos\lambda)(v_0\sin\theta - gt) - (-\omega\sin\lambda)(v_0\cos\theta)\right]\hat{y} + 0\,\hat{z}$$

$$\vec{\omega}\times\vec{v} = +\omega v_0 \cos\lambda \sin\theta - \omega g t\cos\lambda - \omega v_0 \sin\lambda\cos\theta$$

THUS THE CORIOLIS ACCELERATION IS $\left[a_{cor} = -2(\vec{\omega}\times\vec{v}_r)\right]$

$$a_{cor} = 2\omega v_0 \underbrace{\left[\sin\lambda\cos\theta - \cos\lambda\sin\theta\right]}^{\sin(\lambda-\theta)} + (2\omega g\cos\lambda)\,t$$

INTEGRATE TO FIND v_{cor}

$$v_{cor} = 2\omega v_0 \sin(\lambda-\theta)\,t + (\omega g\cos\lambda)\,t^2$$

SO THE $\hat{y}$ DEFLECTION IS

$$y = \omega v_0 \sin(\lambda - \theta)\,t^2 + \tfrac{1}{3}(\omega g\cos\lambda)\,t^3$$

USING z, FIND t $\quad \cancelto{0}{z} = \cancelto{0}{z_0} + (v_0\sin\theta)t - \tfrac{1}{2}gt^2$

$$t_F = \frac{2v_0}{g}\sin\theta$$

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NOTE BOOK

SUBSTITUTING

$$y = \omega v_0 \sin(\lambda - \theta)\left(\frac{2v_0}{g}\sin\theta\right)^2 + $$

$$+ \frac{1}{3}(\omega g \cos\lambda)\left(\frac{2v_0}{g}\sin\theta\right)^3$$

$$y = \frac{4v_0^3\omega}{g^2}\sin(\lambda-\theta)\sin^2\theta - \frac{8v_0^3\omega}{3g^2}\cos\lambda\sin^3\theta$$

$$y = \frac{4v_0^3\omega}{3g^2}\sin^2\theta\left[3\sin(\lambda-\theta) + 2\cos\lambda\sin\theta\right]$$

FOR $v_0 = 800\ m/s$, $\omega = 7.27\times10^{-5}$, $\lambda = 50^\circ$, $\theta = 37^\circ$

$$y = \frac{4(800)^3(7.27\times10^{-5})}{3(9.81)^2}\sin^2(37)\left[3\sin(50-37) + 2(\cos 50)(\sin 37)\right]$$

$$y = (186.78)[1.45] = \boxed{270.6\,m = y}$$

$\rightarrow$ 270.6 m EASTWARD!